求 Laplace Transform

复阻抗

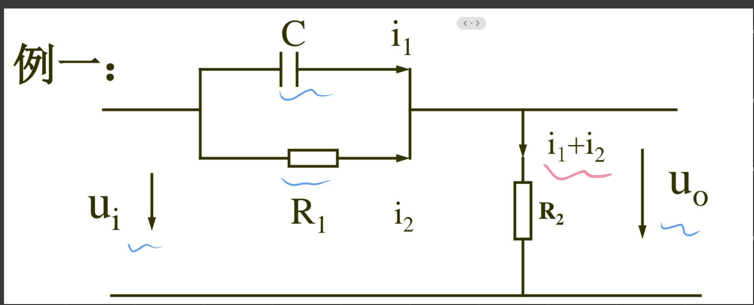
电容 C：$R_C = \frac{1}{Cs}$

电感 L：$R_L = Ls$

开环：$\phi(s) = G(s)\cdot H(s)$

闭环：$\phi(s) = \frac{G(s)}{1+GH}$

Tip：单位负反馈：$GH = \frac{k}{A(s)}$ ⇒ $\phi = \frac{G}{1+GH} = \boxed{\frac{k}{A(s)+k}}$



整体认识框图

对R_2：$U_o = (i_1+i_2)\cdot R_2$ ⇒ $\frac{U_o}{R_2} = i_1+i_2$

对R_1：$i_2 = \frac{U_i-U_o}{R_1}$

对C_1：$i_1 = \frac{U_i-U_o}{1/Cs} = (U_i-U_o)\cdot Cs$

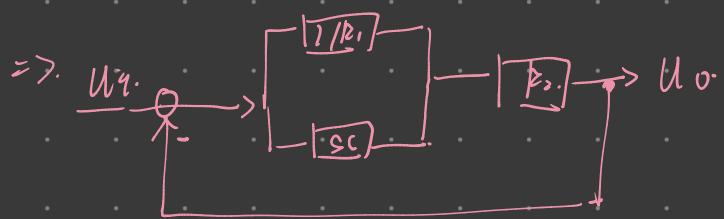
⇒

R_2：$i_1+i_2 \to \boxed{R_2} \to U_o$

R_1：$U_i-U_o \to \boxed{1/R_1} \to i_2$

C_1：$U_i-U_o \to \boxed{Cs} \to i_1$

⇒



方框图的化简变换

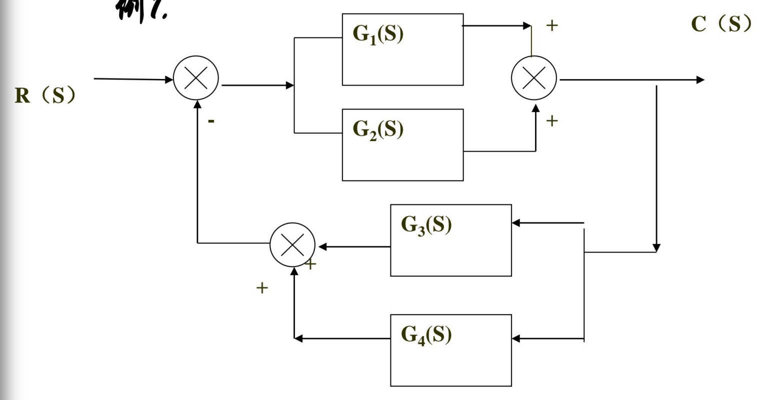
综合点：

后移

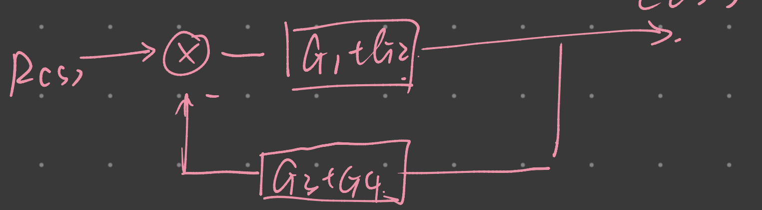
前移

互移

例1.

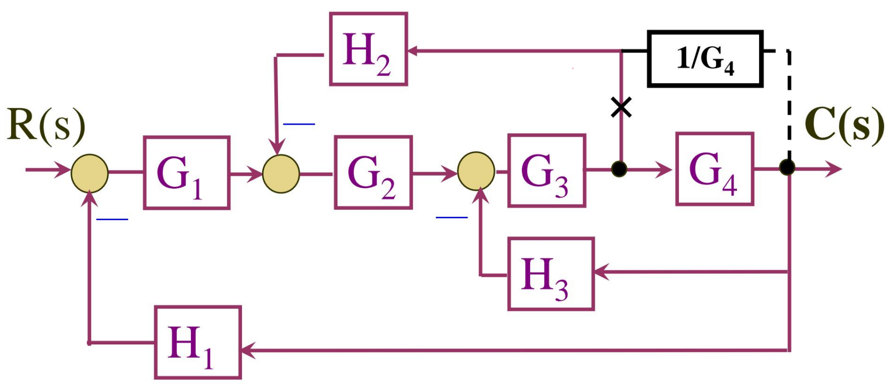


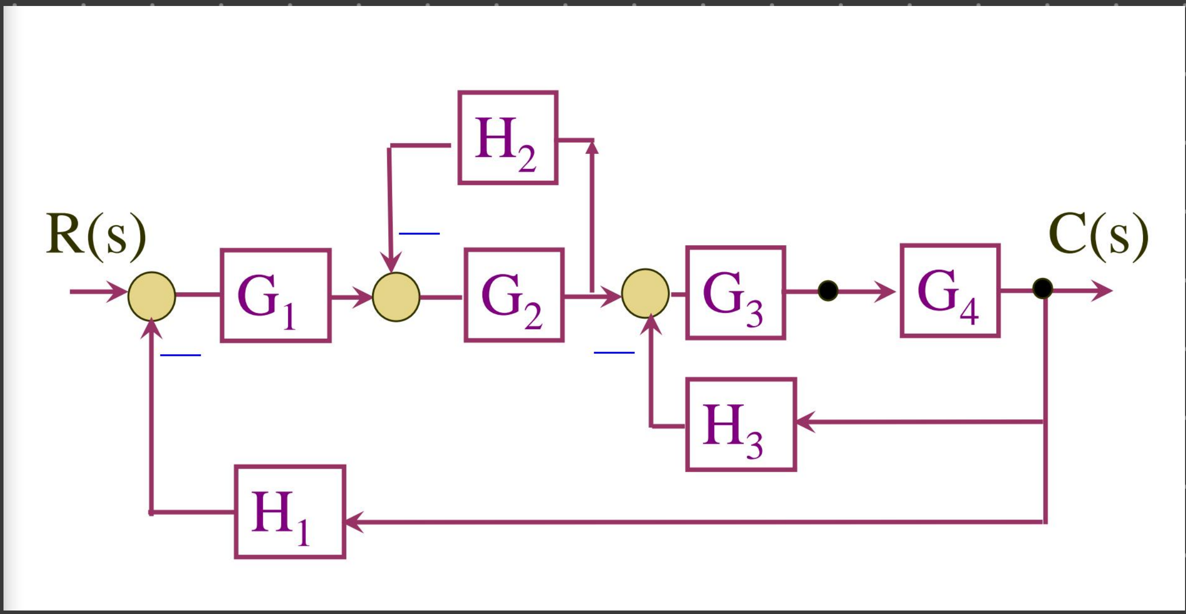
化简:



$$\therefore \Phi = \frac{G}{1+GH} = \frac{G_1+G_2}{1+(G_1+G_2)(G_3+G_4)}$$

例2. 化简下列方块图。



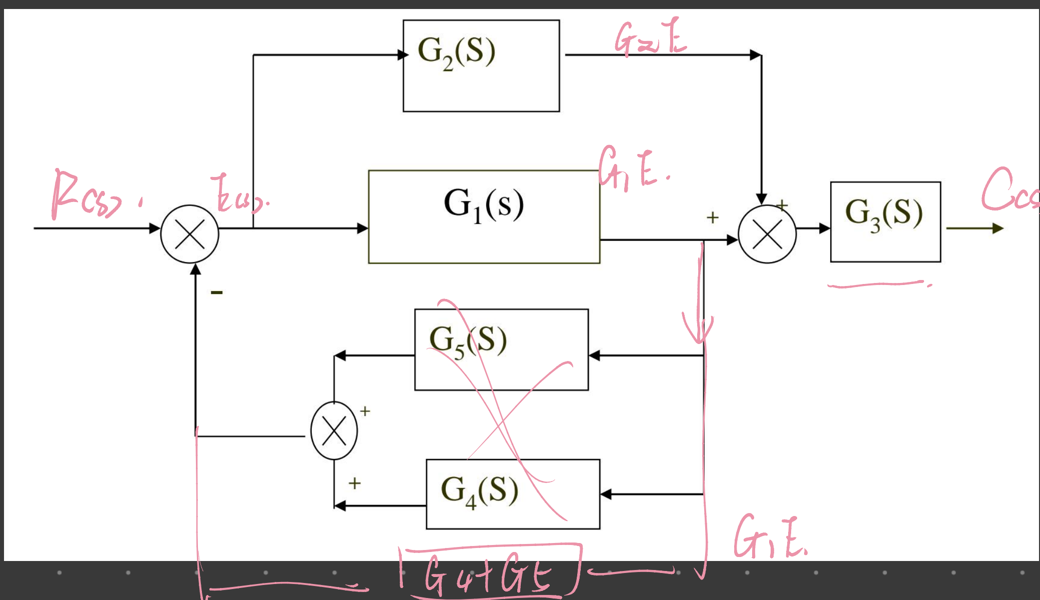


梅逊公式：

$$\Phi = \frac{\text{前向通道传递函数}}{1 \pm \Sigma\,\text{单一回路开环}\Phi(s) \pm \Sigma\,\text{两两不交叉开环}\Phi(s)\text{乘积} \ldots \Sigma\,\text{三三}\ldots}$$

注：分母： $\underbrace{1-\Sigma\Phi(s)}_{\text{负反馈：自带负号，最后为正.}} + \Sigma\Phi(s_1)\cdot\Phi(s_2)$

$$\frac{G_1G_2G_3G_4}{1+G_2H_2+G_3G_4H_3+G_1G_2G_3G_4H_1+G_2G_3G_4H_2H_3}$$



$\Rightarrow \dfrac{(G_1+G_2)G_3}{1+G_1G_4+G_1G_5}\cdot R(s) = C.$

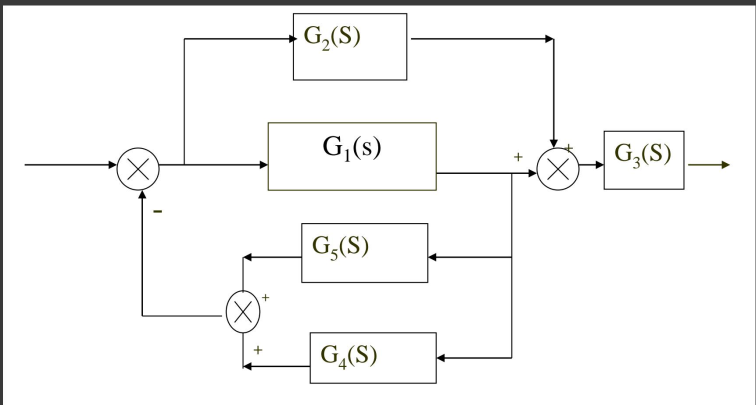
$\therefore \Phi = \dfrac{(G_1+G_2)G_3}{1+G_1G_4+G_1G_5}.$

$R(s) - (G_1E)(G_4+G_5) = E$

$R(s) = E + G_1(G_4+G_5)\cdot E$
$= E(1+G_1G_4+G_1G_5)$

$\therefore E = \dfrac{R(s)}{1+G_1G_4+G_1G_5}$

$(G_1+G_2)\cdot E\cdot G_3 = C$



使用 Mason.

则更简单、

观察到 前向: $(G_1+G_2)\cdot G_3$,

回路: $(G_4+G_5)G_1$

$$\Rightarrow \varphi = \frac{(G_1+G_2)G_3}{1+(G_4G_5)G_1}$$

典型响应.

1、单位阶跃：$h(t)$.

$$C(s) = \Phi(s)\cdot \underline{R(s)}$$

$$h(t) = L^{-1}[C(s)] = L^{-1}[\Phi(s)\cdot \underbrace{R(s)}_{\to \frac{1}{s}}]$$

$$= L^{-1}[\Phi(s)\cdot \frac{1}{s}]$$

2、单位斜坡：$c(t)$

$$\Rightarrow c(t) = L^{-1}[\Phi(s)\cdot \frac{1}{s^2}]$$

3、单位脉冲：$g(t)$

$$g(t) = L^{-1}[\Phi(s)]$$

另：$\frac{d(h(t))}{dt} = g(t)$.

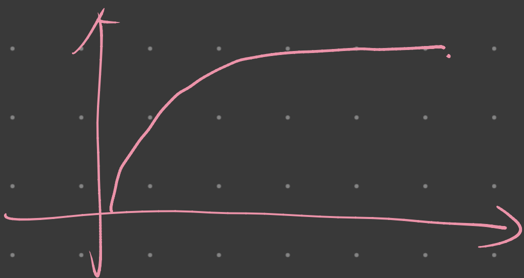
稳：M_p

准：e_{ss}，

快：t_s

一阶系统：通式： → 放大系数

$\Phi(s) = \frac{C(s)}{R(s)} = \frac{1}{Ts+1}$ 形



阶跃响应：

$t_s = 3T \rightarrow 5\%$

$t_s = 4T \rightarrow 2\%$

例: 结构图如下:

是一阶系统,

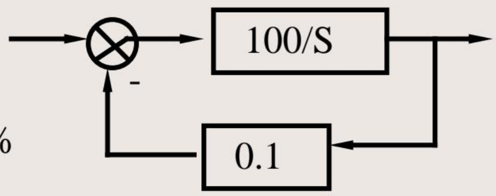
对快速性的要求是,

在0.1秒之内进入 ±5%

误差带(即 t_s=0.1)

看是否满足要求?如不能,

一个怎样调整 反馈系数 H



$\Phi = \frac{1}{Ts+1}$

$\Phi = \frac{G}{1+GH} = \frac{100/s}{1+\frac{10}{s}} = \frac{100}{s+10} = \frac{10}{0.1s+1} \Rightarrow T=0.1$

$t_s = 0.1 = 3T$

$t_s = 3T = 0.3s > 0.1$ ∴不行.

$\Rightarrow \Phi = \frac{G}{1+GH} = \frac{100/s}{1+100H/s} = \frac{100}{s+100H} = \frac{\frac{1}{H}}{\frac{1}{100H}s+1}$

$T = \frac{1}{100H} = \frac{0.1}{3}$

$\therefore H = \frac{3}{10} = 0.3$

二阶系统:

$$\frac{C(s)}{R(s)} = \Phi(s) = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n + \omega_n^2}$$

$\boxed{\text{单位阶跃:}}$

$$s^2 + 2\zeta\omega_n + \omega_n^2 = 0$$

$$-\zeta\omega_n \pm \omega_n\sqrt{\zeta^2 - 1}$$

$\Rightarrow$ ① $\zeta > 1$ 过阻尼 $\Rightarrow$ 稳 $\longrightarrow$ $t_s = \frac{4.5\zeta}{\omega_n}$

② $\zeta = 1$ 临 $\Rightarrow$ 不稳 $\longrightarrow$ $t_s = \frac{4.75}{\omega_n}$

③ $\zeta < 1$ 负~ $\Rightarrow$ 不稳. $\longrightarrow$ $t_s = \begin{cases} \frac{3.2}{\zeta\omega_n} & (0 < \zeta < 0.69) \\ \frac{4.5\zeta}{\omega_n} & \zeta > 0.69 \end{cases}$

查表.

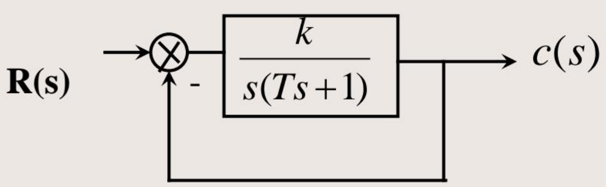
$\zeta = 0.707$ 时.

$t_s = \frac{3}{\omega_n}$

$M_p\% = 4.3$

$e_{ss} = 0$

例：小功率随动系统，结构图：



T=0.1秒，k是开环增益(Open-loop gain)。

（开环增益：系统开环传递系数中，将最低阶分子、分母的系数换算成1后的放大系数。

$$G(s)=\frac{6(s+2)}{9s^2+6s+3}=\frac{4(0.5s+1)}{3s^2+2s+1}$$ 则k=4

要求，系统单位阶跃响应无超调，t_s=1秒（5%误差带）求k.

$M_p=0 \Rightarrow$ 过阻尼或临界阻尼.

$$\Phi=\frac{G}{1+G(s)}$$

$$=\frac{k}{s(Ts+1)+k}=\frac{k}{Ts^2+s+k}=\frac{10k}{s^2+10s+10k}$$

$$=\frac{\omega_n^2}{s^2+2\zeta\omega_n s+\omega_n^2}$$

$$\begin{cases}\omega_n^2=10k \\ 2\zeta\omega_n=10 \\ t_s=\frac{4.5\zeta}{\omega_n}=1s\end{cases}$$

$\omega_n=\sqrt{10k}$

$\Rightarrow \zeta=\frac{5}{\omega_n}=\frac{5}{\sqrt{10k}}$

$\Rightarrow \frac{4.5\times5/\sqrt{10k}}{\sqrt{10k}}=\frac{22.5}{10k}=1s$

$\therefore k=2.25$

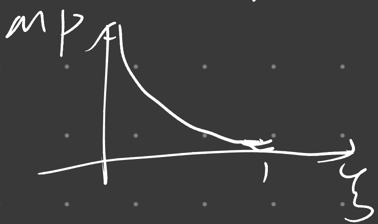
欠阻尼

$$s_{1,2} = -\xi\omega_n \pm \omega_n \cdot \sqrt{\xi^2 - 1}$$

$$= -\sigma \pm j\omega_d.$$

$$\omega_d = \omega_n \cdot \sqrt{1-\xi^2}.$$

① 超调量: $M_p\% = e^{\frac{-\pi\xi}{\sqrt{1-\xi^2}}} \times 100\%$



② 调节时间 t_s

$\xi = 0.707$ 时为最佳阻尼比

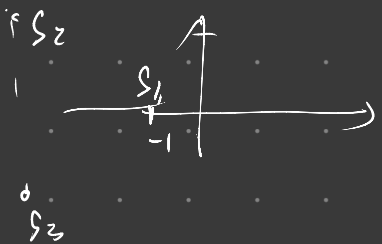
高阶估算：

① 主导极点：离虚轴近

$$\Phi(S)=\frac{100}{(S+1)(S^2+10S+100)}$$

$D(s)=(s+1)(s^2+10s+100)$

$s_1=-1 \quad s_{2,3}=-5\pm\sqrt{-300}/2.$



主导化简.

$\Rightarrow \Phi(s)\approx\frac{100}{s+1}$

② 偶极子法：

看零点。

稳定性

$$L^{-1}[C(s)] = c(t) = \sum B_{j} \cdot e^{s_j t} + \sum A_i e^{s_i t}$$

稳态分量　　暂态分量

$\Downarrow$

$\lim\limits_{t\to\infty} \sum A_i e^{s_i t} \to 0$

稳定.

$$\Phi(s) = \frac{M(s)}{D(s)}$$

$D(s)$ 为特征方程.

s_i 为特征根.

① 实根 $s_i = \sigma_i$
$$\begin{cases} \sigma > 0 & \text{不稳} \\ \sigma = 0 & \text{临界} \\ \sigma < 0 & \text{稳.} \end{cases}$$

② 复根 $s_i = \sigma_i \pm j\omega_i$

$\Rightarrow$ $\sigma < 0$ 时 稳

$\sigma > 0$ 时 不稳

例：判断下列单位负反馈系统的稳定性：

开环传递函数为 $G(s)=\dfrac{k}{(s+10)(s-4)}$

$$\Phi = \frac{G}{1+GH} = \frac{k}{(s+10)(s-4)+k} = \frac{k}{s^2+6s-40+k}$$

$$\Rightarrow s_{1,2} = \frac{-6 \pm \sqrt{36+160-4k}}{2} = -3 \pm \sqrt{49-k}$$

分类的实部为负 判稳

$$\sqrt{49-k} < 3 \Rightarrow k > 0$$

∴ $k>40$ 时稳、 $k \le 40$ 不稳

⊛ 代数稳定判据、

劳斯判据

$$D(s) = a_0 s^n + a_1 s^{n-1} + a_2 s^{n-2} + \cdots + a_n s^0$$

劳斯表

s^n	a_0	a_2	a_4
s^{n-1}	a_1	a_3	a_5
s^{n-2}	b_1	b_2	b_3
s^{n-3}	c_1	c_2	c_3

trick: 交叉相乘相减

$$b_1 = \frac{a_1 a_2 - a_0 a_3}{a_1} \qquad c_1 = \frac{b_1 a_3 - a_1 b_2}{b_1}$$

$$b_2 = \frac{a_1 a_4 - a_0 a_5}{a_1} \qquad c_2 = \frac{b_1 a_5 - a_1 b_3}{b_1}$$

例1： D(s)=S³+3S²+S+2

$$\underset{a_0}{1S^3} + \underset{a_1}{3S^2} + \underset{a_2}{1S} + \underset{a_3}{2}$$

$$\begin{array}{c|cc} s^3 & 1 & 1 \\ s^2 & 3 & 2 \\ s^1 & \frac{1}{3} & 0 \\ s^0 & 2 & 0 \end{array}$$

$$\begin{cases} a_i > 0 \\ x_1 > 0 \end{cases} \Rightarrow \text{稳定.}$$

例2：单位负反馈系统, 求K的取值范围（稳定域）

$$G(s) = \frac{K}{S(0.1S+1)(0.25S+1)}$$

$$D(s) = s(0.1s+1)(0.25s+1)+K$$

$$= \underset{a_0}{0.025s^3} + \underset{a_1}{0.35s^2} + \underset{a_2}{s} + \underset{a_3}{K}$$

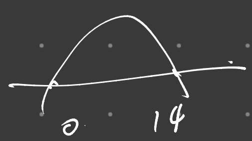
① $K>0$

② 劳斯表

$$\begin{array}{c|cc} s^3 & 0.025 & 1 \\ s^2 & 0.35 & K \\ s^1 & \frac{0.35-0.025K}{0.35} & 0 \\ s^0 & \frac{0.35K-0.025K^2}{0.35^2} & 0 \end{array}$$

$$\Rightarrow \begin{cases} 0.35-0.025K>0 \\ (0.35-0.025K)K>0 \end{cases}$$

$$\Rightarrow \begin{cases} K<14 \\ 0<K<14 \end{cases}$$



$\therefore$ $0<K<14$ 时稳定.

稳态误差分析

$$e_{ss} = \lim_{t\to\infty} e(t)$$

$$= \lim_{t\to\infty} (r(t) - b(t)) = \lim_{s\to 0} sE(s)$$

①仅 $r(t)$ 作用，$n(t)=0$

$$\Rightarrow e_{sr} = \lim_{s\to 0} sR(s) \cdot \frac{1}{1+G_1G_2H}$$

$$E(s) = \frac{1}{1+GH} \cdot R(s)$$

$\Rightarrow$ 误差传递函数 $\Phi_{er} = \dfrac{1}{1+GH}$

②型别：

看开环传递 GH

$$GH = \frac{\quad}{s^{\nu} \prod (\tau_i s+1)}$$

$$\begin{cases} \nu = 0 & \text{0型} \\ \nu = 1 & \text{I型} \\ \nu = 2 & \text{II型} \end{cases}$$

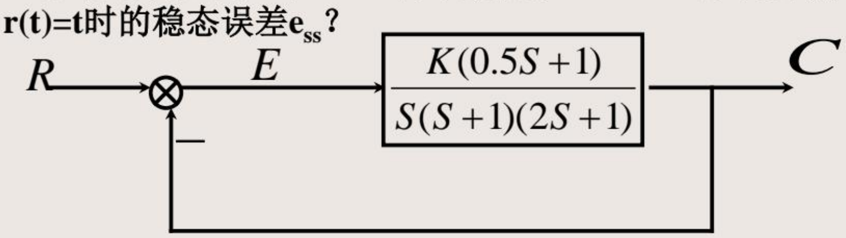
$\Rightarrow$ 型别表：

不同系统型别及输入情况下的 e_{sr} 值

Input	type			
	0	1	2	3
Unit step	$\frac{1}{1+K}$	0	0	0
Unit ramp	∞	$\frac{1}{K}$	0	0
Unit parabolic	∞	∞	$\frac{1}{K}$	0

K:开环增益

例：系统结构如图，求（1）系统稳定性，（2）系统的输入 $r(t)=t$ 时的稳态误差 e_{ss}？



(1) 判稳：

$$\Phi=\frac{G}{1+GH}=\frac{k(0.5s+1)}{s(s+1)(2s+1)+k(0.5s+1)}=\frac{k(0.5s+1)}{\underset{a_0}{2s^3}+\underset{a_1}{3s^2}+\underset{a_2}{(1+0.5k)s}+\underset{a_3}{k}}$$

$\Rightarrow$ ① $\begin{cases}1+0.5k>0\\k>0\end{cases}$ $\begin{cases}k>-2\\k>0\end{cases}$

②

s^3	2	$1+0.5k$
s^2	3	k
s^1	$\frac{3-0.5k}{3}$	0
s^0	$\frac{3(3-0.5k)k}{3-0.5k}$	0

$\therefore$ 综上 $0<k<6$

$\Rightarrow \begin{cases}3-0.5k>0\\k>0\end{cases} \Rightarrow \begin{cases}k<6\\k>0\end{cases}$

(2) 稳态误差，$r(t)=t$，斜坡.

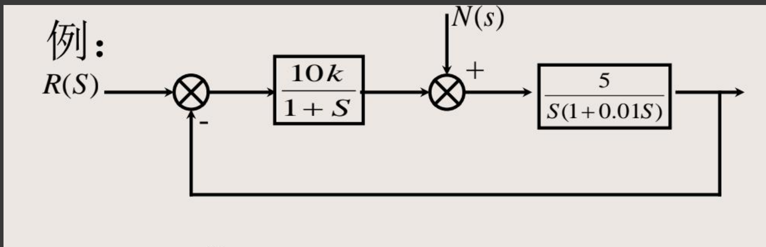
开环 $\Phi=GH=\frac{k(0.5s+1)}{s(s+1)(2s+1)}$ $\Rightarrow$ Ⅰ型

$$e_{ss}=\frac{1}{K_v}=\frac{1}{\lim\limits_{s\to0}s\cdot G(s)\cdot H(s)}=\frac{1}{\lim\limits_{s\to0}\frac{k(0.5s+1)}{(s+1)(2s+1)}}=\frac{1}{k}$$

另一种思路：按定义：$e_{ss}=\lim\limits_{s\to0}sE(s)=\lim\limits_{s\to0}R(s)\cdot\Phi_{er}(s)$

$$=\lim_{s\to0}R(s)\cdot\frac{1}{1+GH}=\lim_{s\to0}\frac{1}{s^2}\cdot\frac{1}{1+(\ \ )}\Rightarrow\frac{1}{k}.$$

二、存在



n(t)=4,求e_{sn}

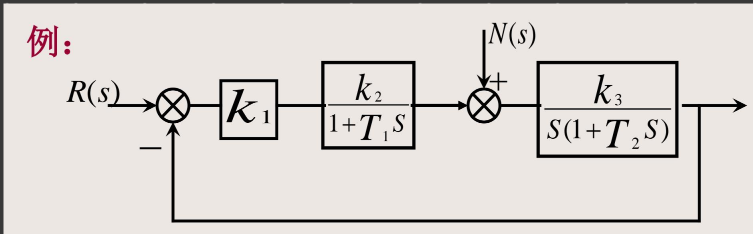
$\hookrightarrow N(s) = \frac{4}{s}$

$\Phi_{sn} = -\frac{G_2H}{1+G_1G_2H}$

$$e_{sn} = \lim_{s\to 0} s \cdot N(s) \left(-\frac{G_2H}{1+G_1G_2H}\right)$$

$$= \lim_{s\to 0} s \cdot \frac{4}{s} \cdot \frac{\frac{-5}{s(1+0.01s)}}{1+\frac{10k}{1+s}\cdot\frac{5}{s(1+0.01s)}} = \lim_{s\to 0} 4 \cdot \frac{-5}{s(1+0.01s)+\frac{50k}{1+s}} = \frac{-20}{50k}$$

$$= -\frac{2}{5k}.$$



K_2=10，K_3=5，T_1=1，T_2=0.01

r(t)=2+t， n(t)=4

要求： e_{ss}<=0.05， 求K_1